Topic B
Division as an Unknown Factor Problem

3.OA.2, 3.OA.6, 3.OA.3, 3.OA.4

Focus Standard: 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

3.OA.6 Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Instructional Days: 3
Coherence -Links from: G2–M6 Foundations of Multiplication and Division
-Links to: G4–M3 Using Place Value Understanding and Properties of Operations to Perform Multi-Digit Multiplication and Division

The study of factors links Topics A and B; Topic B extends the study to division. Students continue to use a variety of factors in this topic as the emphasis in these lessons rests on conceptually understanding division and learning to interpret problems by writing division expressions. Students understand division as an unknown factor problem, and in Lessons 4 and 5 relate the meaning of the unknown in division to the size of groups and the number of groups, respectively. They work through word problems that help give meaning through context, and then analyze more abstract drawings.

In Lesson 6, students explore division in the context of the array model, interpreting arrays by writing division sentences. Through the array students relate the unknown factor in multiplication to the quotient in division. They use arrays to write multiplication sentences and find unknown factors, then write division sentences where the quotient represents the same as the unknown factor. By the end of this topic students use the vocabulary quotient and unknown factor, and discussion moves toward solidifying understanding of the relationship between multiplication and division.
## A Teaching Sequence Towards Mastery of Division as an Unknown Factor Problem

<table>
<thead>
<tr>
<th>Objective</th>
<th>Description</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective 1:</td>
<td>Understand the meaning of the unknown as the size of the group in division.</td>
<td>(Lesson 4)</td>
</tr>
<tr>
<td>Objective 2:</td>
<td>Understand the meaning of the unknown as the number of groups in division.</td>
<td>(Lesson 5)</td>
</tr>
<tr>
<td>Objective 3:</td>
<td>Interpret the unknown in division using the array model.</td>
<td>(Lesson 6)</td>
</tr>
</tbody>
</table>
Lesson 4

Objective: Understand the meaning of the unknown as the size of the group in division.

Suggested Lesson Structure

- Fluency Practice (15 minutes)
- Application Problem (5 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (15 minutes)

- Repeated Addition as Multiplication 3.OA.1 (10 minutes)
- Group Counting 3.OA.1 (3 minutes)
- Array Multiplication 3.OA.1 (2 minutes)

Sprint: Repeated Addition as Multiplication (10 minutes)

Materials: (S) Repeated Addition as Multiplication Sprint

Note: Students relate repeated addition to multiplication. This reviews Topic A’s objectives. See Directions for Administration of Sprints in Lesson 2.

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos and threes in this activity anticipates work with those factors in this lesson.

T: Let’s count by twos. (Direct students to count forward and backward to 20, periodically changing directions, e.g., 2, 4, 6, 8, 10, 8, 10, 12, 10, 12, 14, 16, 18, 20, 18, 20, 18, 16, 14, 12, 10, 12, 10, 8, 10, 8, 6, 4, 2, 0.)

T: Let’s count by threes. (Direct students to count forward and backward to 24, periodically changing directions. Emphasize the 9 to 12 and 18 to 21 transitions, e.g., 3, 6, 9, 12, 9, 12, 9, 12, 15, 18, 21, 18, 21, 18, 21, 18, 15, 12, 15, 12, 9, 12, 9, 6, 3, 0.)
Lesson 4:
Understand the meaning of the unknown as the size of the group in division.

Date: 5/6/13

Array Multiplication (2 minutes)

Materials: (S) Personal white boards.

Note: This activity reviews Topic A’s objectives. Students directly relate repeated addition to multiplication. They interpret products using the array.

T: (Project a picture array with 3 groups of 2 circled.) Say the repeated addition sentence.
S: 2 + 2 + 2 = 6.
T: (Write 3 × ____ = ____.) On your personal boards, complete the multiplication sentence.
S: (Write 3 × 2 = 6.)

Continue with possible sequence: 4 groups of 10, 3 groups of 4, 7 groups of 3, and 8 groups of 2.

Application Problem (5 minutes)

The student council holds a meeting in Mr. Chang’s classroom. They arrange the chairs in 3 rows of 5. How many chairs are used in all? Use the RDW process.

Note: This problem reviews relating multiplication to the array model from Lesson 2. Students may choose to solve by drawing an array (Lesson 2) or a number bond (Lesson 3) where each part represents the amount of chairs in each row.

Concept Development (30 minutes)

Materials: (S) Personal boards, 18 counters per student

Problem 1

Concrete to abstract: Division as fair sharing. Relating the answer to the unknown factor.

T: Yesterday our neighbor Mr. Ziegler bought a new pack of 18 markers. He wanted to share them with me, so this morning he divided them into 2 equal groups. Now I have a bunch of new markers for making our charts! Do you want to know how many he gave me?
T: What are we trying to find, the number of groups or the size of the group?
S: The size of the group.
T: Your 18 counters represent the markers. Divide your 18 counters into 2 equal groups, by giving one to Mr. Z,
Lesson 4: Understand the meaning of the unknown as the size of the group in division.

Date: 5/6/13
Problem 2

Pictorial to abstract: Analyzing a picture to write a division equation wherein the solution tells the size of the group.

T: (Project or draw the following image.) This is how Diana arranges her star stickers.

```
* * * *
* * * *
* * * *
```

T: What does the 12 represent in the picture?
S: The total number of Diana’s star stickers.
T: What does the 3 represent?
S: The number of equal groups.
T: What does the 4 represent?
S: The size of each group.
T: Write an equation to represent Diana’s stickers where the answer represents the size of the group.
S: (Write 12 ÷ 3 = 4.)
T: (Write 12 ÷ 3 = 4 and 12 ÷ 4 = 3 on the board, even if students have written the correct number sentence.) What is the difference between these division sentences?
S: In the first one the answer represents the size of each group. In the second one the answer represents the number of groups.
T: If we’re writing a division sentence where the answer represents the size of the group, then which equation should we use?
S: 12 ÷ 3 = 4.

Problem 3

Abstract to pictorial: 10 ÷ 2, analyze equations for the meaning of the solution and represent the equation with a drawing.

Write 8 ÷ 4 = ___.

T: If 8 is the total and 4 is the number of groups, then what does the unknown factor represent?
S: The size of the groups!
T: Draw a picture on your board to go with my division sentence. Use your picture to help you find the unknown factor, then write the complete equation.
S: (Draw various pictures that show 8 ÷ 4, then write 8 ÷ 4 = 2)

Repeat the process with 10 ÷ 2. As you design examples, keep in mind that Lesson 5 introduces students to division where the unknown factor represents the number of groups.
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Understand the meaning of the unknown as the size of the group in division.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Ask students to share their number sentences for Problem 9. Because of the way the question is worded, answers will likely include 15 ÷ 5 = 3 (answer is the size of the group) and 15 ÷ 3 = 5 (answer is the number of groups). This presents an opportunity to begin a discussion in which students compare the equations by analyzing the meaning of the factors.
- Guide students to articulate the similarities and differences between multiplication and division, so that they are clear that division is used to find totals: total number of groups or objects in a group. This can also be thought of as a known factor and an unknown factor.
- Review other vocabulary words and phrases, such as unknown factor and divided by.
Lesson 4: Understand the meaning of the unknown as the size of the group in division.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

This lesson does not use the word division in the dialogue. However, the word does appear on the Problem Set. For some students it may be necessary to preview the word division. You may want to refer them back to the equal groups activity from Lesson 3 and fair sharing with Mr. Ziegler’s markers from Problem 1 to connect it to their understanding of the concept of divide.
Lesson 5

Objective: Understand the meaning of the unknown as the number of groups in division.

Suggested Lesson Structure

- **Fluency Practice**  (8 minutes)
- **Application Problem**  (7 minutes)
- **Concept Development**  (35 minutes)
- **Student Debrief**  (10 minutes)

**Total Time**  (60 minutes)

**Fluency Practice**  (8 minutes)

- Group Counting  **3.OA.1**  (4 minutes)
- Divide Equal Groups  **3.OA.2**  (4 minutes)

**Group Counting**  (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos and threes in this activity supports work with those factors in Topic B.

- T: Let's count by twos. (Direct students to count forward and backward to 20, emphasizing the 8 to 10, 10 to 12, and 18 to 20 transitions.)
- T: Let's count by threes. (Direct students to count forward and backward to 27, changing directions. Emphasize the 9 to 12 and 18 to 21 transitions.)

Record the count-by three up to 24 to use later in the lesson.

**Divide Equal Groups**  (4 minutes)

Materials:  (S) Personal white boards.

Note: Students directly relate repeated addition to division. They interpret the number of groups as the unknown in division. This activity anticipates the lesson objective.

- T: (Project an array with 2 groups of 5 circled.) How many groups are circled?
- S: 2.
- T: How many are in each group?
- S: 5.
Lesson 5: Understand the meaning of the unknown as the number of groups in division.

Date: 5/6/13

T: Say the total as a repeated addition sentence.
S: $5 + 5 = 10$.
T: Write a division sentence for 10 divided into 2 equal groups.
S: (Write $10 \div 2 = 5$.)

Continue with possible sequence 4 groups of 2, 3 groups of 4 and 2 groups of 6.

Application Problem (7 minutes)

Stacey has 18 bracelets. After she organizes the bracelets by color she has 3 equal groups. How many bracelets are in each group?

Note: This problem reviews the meaning of the unknown as the size of the group in division from Lesson 4. It also provides a comparison to Cynthia’s party problem in Problem 1 of the concept development, where the unknown represents the number of groups in division.

Concept Development (35 minutes)

Materials: (S) Personal white boards, 18 counters for each student, student work from application problem

Problem 1

Concrete to abstract: Division as fair-share, the unknown as the number of groups.

T: Next weekend my friend Cynthia is having a party. 18 people are coming. I told her I’d help her setup tables. We know that 6 people can sit at each table, but we’re not sure how many tables we’ll need. Turn and talk with your partner. What information do Cynthia and I already have?
S: They know the total number of people. It’s 18. → Yeah, and they know how many people are sitting together, 6. That’s the size of the group.
T: What information don’t we know?
S: You don’t know how many tables. → Tables are like groups. You don’t know the number of groups.
T: Let’s use counters to show the problem and check our thinking. Each of you has 18 counters, 1 for each person coming to the party. Put them into groups of 6.
S: (Make groups of 6.)
T: Do you still agree we know the total and the size of each group?
S: Yes!
T: Looking at our models, what else do we now know?
S: We know there are 3 groups. → So that means Cynthia needs 3 tables to fit everyone.
T: (Write $18 \div 6 = 3$ on the board.) How does this equation relate to the problem we just solved?
S: The equation shows that we divided. → We knew the total, 18 people. We divided them into groups with 6 people. Then we figured out that meant 3 groups of people. → We divided the total by the size of the group and found the number of groups.
Lesson 5

Understand the meaning of the unknown as the number of groups in division.

Date: 5/6/13

T: Look back at your work from today’s application problem. With your partner, compare the steps you took to solve both the bracelet problem and the party problem. Notice the equations too.

S: For the bracelets I drew circles to show 3 groups first. Then I shared the bracelets between the groups. In the party problem we put the people in groups of 6 first. Then we found how many groups. The 6 and 3 switched places. That’s because in the bracelet problem we had to find the size of the groups, and in the party problem we had to find the number of groups.

T: I’m hearing you notice that the unknown was different in each problem. We divide when we want to find the size of the groups or the number of groups.

Repeat the process using 14 ÷ 7 = ___, without a story context. Focus on 7 being the size of the groups. Match the equation to a number bond.

Problem 2

Relate finding the number of groups to counting by the divisor.

T: Cynthia plans to buy 15 burgers. 3 burgers come in each pack. How many packs should she buy? Whisper to your partner what the numbers 15 and 3 represent in this problem.

S: 15 is the total number of burgers. 3 is the number of burgers in a pack.

T: Is the unknown the number of groups or the size of the group?

S: The number of groups.

T: On your board write the division sentence you would use to find how many packs to buy.

S: (Write 15 ÷ 3 = ____.)

T: Let’s draw to find out how many packs Cynthia needs.

S: (Students draw.)

T: How many packs did Cynthia need?

S: 5 packs.

T: 15 ÷ 3 is?

S: 5.

T: Let’s write the total number of burgers under each pack. How many total burgers does she have in one pack?

S: 3 burgers.

T: In two packs?

S: 6 burgers (repeat the process up to 15).

T: Let’s read our numbers.

S: 3, 6, 9, 12, 15.

T: Why did we stop at 15?

S: Because Cynthia only needs 15 burgers.

NOTES ON MULTIPLE MEANS FOR ENGAGEMENT:

Students may need help remembering to relate the number of groups to the more familiar parts shown by the number bond. You may want to have them work in partners to draw, or scaffold the process for the whole group by walking them through it.
Lesson 5
Understand the meaning of the unknown as the number of groups in division.

Date: 5/6/13

T: What connection can you make between this problem and our fluency (indicate the count by three series from earlier)?
S: It’s like counting by three.
T: Yes. Each time we add a group, we add a three.
T: Count by threes with me and track the number of threes on your fingers.
S: 3, 6, 9, 12, 15. (Start with a closed fist and stick out one finger each time you say a three.)
T: How many threes did we count?
S: 5.
T: Skip-counting also shows us that Cynthia needs 5 packs.

Repeat the process with $21 \div 3 = \_\_\_$ and $14 \div 2 = \_\_\_$, not in a story context.

T: A count-by can be a quick way to solve division problems when we need to find the number of equal groups. Especially if we have a big total like 21!

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Understand the meaning of the unknown as the number of groups in division.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Activity Problem Set. They should check work by comparing answers with a partner before going over answers as a class.
Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Review the relationship of multiplication to division. Guide students to observe that division is used to find either factor—the unknown can be the size of groups (learned yesterday) or the number of groups (learned today)
- Review the vocabulary groups of.
- Practice using the count-by strategy to solve Problem Set problem 5.
- How is a number bond diagram different than a drawing representing a count-by?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
Lesson 6

Objective: Interpret the unknown in division using the array model.

Suggested Lesson Structure

- Fluency Practice (9 minutes)
- Application Problem (6 minutes)
- Concept Development (35 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (9 minutes)

- Group Counting 3.OA.1 (4 minutes)
- Divide Equal Groups 3.OA.2 (5 minutes)

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos and threes in this activity supports work with those factors in Topic B.

T: Let’s count by twos. (Direct students to count forward and backward to 20, emphasizing the 8 to 10, 10 to 12, and 18 to 20 transitions.)

T: Let’s count by threes. (Direct students to count forward and backward to 30, periodically changing directions. Emphasize the 9 to 12, 18 to 21, and 27 to 30 transitions.)

Divide Equal Groups (5 minutes)

Materials: (S) Personal white boards

Note: Students directly relate repeated addition to division. They interpret the unknown in division. This activity bridges Lessons 5 and 6.

T: (Project an array with 3 groups of 5 circled.) Say the total as a repeated addition sentence.

S: 5 + 5 + 5 = 15.

T: Write a division sentence for 15 divided into 3 equal groups.

S: (Write 15 ÷ 3 = 5.)

Continue with possible sequence: 5 groups of 3, 4 groups of 3, 3 groups of 4, 9 groups of 2, and 2 groups of 9.

Alternate between students writing division sentences where the quotient represents either the number of objects in a group or the number of groups.
Application Problem (6 minutes)

20 children play a game. There are 5 children on each team. How many teams play the game? Write a division sentence to represent the problem.

Note: This problem reviews division from Lesson 5 where the unknown represents the number of groups. It also leads into problem 1 of today’s lesson as it relates division to the array model.

Concept Development (35 minutes)

Materials: (S) Personal white boards, application problem

Problem 1

Relate division to an array model.

T: (Draw an array representing the application problem.)

Have students analyze the array and describe the following relationships:

- Total number of children and total number of dots
- Number of children on each team and number of dots in each row
- Number of teams and number of rows

Repeat the process with the following suggested examples. This time guide students to draw the array from the division sentences below. Alternate between having the quotient represent the size of the groups and the number of groups.

- $8 \div 2 = 4$
- $18 \div 6 = 3$

NOTES TO THE TEACHER ON ARRAYS:

Problem 1 in this lesson introduces students to relating division to an array model for the first time. In Lesson 2, students relate the rows in an array to the number of equal groups and the number of dots in each row to the size the group. The same concept applies for division arrays, but now the problems begin with the total number.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Some students may benefit from working with a partner. They may underline each row to literally show division and circle each row to show the size of each group. They should explain each step they take. This may be particularly helpful for children who prefer visual or kinesthetic practice along with auditory.
Problem 2

Use an array to relate the unknown factor in multiplication to the quotient in division.

T: Draw an array that shows the division sentence $15 \div 3 = 5$, where the *quotient*—that means the answer—represents the size of the groups.

S: (Draw array below.)

```
  ● ● ● ● ●
  ● ● ● ● ●
  ● ● ● ● ●
```

T: Now write both a division and a multiplication sentence for the array.

S: (Write $15 \div 3 = 5$, $3 \times 5 = 15$.)

T: Where do you find the quotient in our multiplication sentence?

S: It’s the second number. → It’s the size of the groups. → It’s a factor.

T: Circle the size of the groups in both problems.

S: (Circle the 5 in both problems.)

Repeat the process with the following suggested examples. Alternate between having the quotient represent the size of the groups and the number of groups.

- 4 rows of 2
- 7 rows of 3

T: Use our equations to explain to your partner how the factors in a multiplication problem can help you find the quotient in division.

---

Problem 3

Relate multiplication and division.

T: (Write ___ $\times 3 = 24$ on the board.) Skip-count and track the number of threes to solve.

S: 3, 6, 9, 12, 15, 18, 21, 24. (Write 8’ to complete the equation.)

T: How many threes make 24? Answer in a complete sentence.

S: 8 threes make 24.

T: Write a related division sentence where the quotient represents the unknown factor.

S: (Write $24 \div 3 = 8$.)

T: 24 divided into threes makes how many groups? Answer in a complete sentence.

S: 24 divided into threes makes 8 groups.

T: How are the unknown factor and the quotient related in these equations?

S: The unknown factor represents the same as the quotient.

---
Repeat the process with the following suggested examples:

- 2 × ____ = 18 and 18 ÷ 2 = ____
- ____ × 9 = 27 and 27 ÷ 9 = ____

T: (Write __ × 3 = 24 and 24 ÷ 3 = __.) True or false: Both equations ask “How many threes are in 24?”

S: They look different, but they mean the same thing. In both we’re talking about 8 groups of 3 and a total of 24. So it’s true. → The quotient in a division problem is like finding the unknown factor in a multiplication problem.

**Problem Set (10 minutes)**

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

**Student Debrief (10 minutes)**

**Lesson Objective:** Interpret the unknown in division using the array model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Analyze the 4 number sentences in Problem 3. Compare the multiplication and division sentences, noticing the differences in how the problem is represented by each one.
Lesson 6: Interpret the unknown in division using the array model.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students’ understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

- How do arrays represent both multiplication and division?
- Based on your observation of arrays, what do multiplication and division have in common?
- What is the relationship between the quotient in division and the unknown factor in a related multiplication sentence?